



**Calhoun: The NPS Institutional Archive**  
**DSpace Repository**

---

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

---

1989-03

# The dual decomposition method and its application to an interdicted network.

Gannon, Timothy Paul

Monterey, California. Naval Postgraduate School

---

<http://hdl.handle.net/10945/27002>

---

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

*Downloaded from NPS Archive: Calhoun*



<http://www.nps.edu/library>

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

**Dudley Knox Library / Naval Postgraduate School**  
**411 Dyer Road / 1 University Circle**  
**Monterey, California USA 93943**









# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

G14397

THE DUAL DECOMPOSITION METHOD AND ITS  
APPLICATION TO AN INTERDICTED NETWORK

by

Timothy P. Gannon

March 1989

Thesis Advisor: Michael P. Bailey

Approved for public release; distribution is unlimited

T241927



## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION <b>Unclassified</b>			1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION / AVAILABILITY OF REPORT <b>Approved for public release; distribution is unlimited</b>		
2b DECLASSIFICATION / DOWNGRADING SCHEDULE			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
4 PERFORMING ORGANIZATION REPORT NUMBER(S)			5a NAME OF MONITORING ORGANIZATION <b>Naval Postgraduate School</b>		
6a NAME OF PERFORMING ORGANIZATION <b>Naval Postgraduate School</b>		6b OFFICE SYMBOL (if applicable) <b>55</b>	7a NAME OF MONITORING ORGANIZATION <b>Naval Postgraduate School</b>		
6c ADDRESS (City, State, and ZIP Code) <b>Monterey, California 93943-5000</b>			7b ADDRESS (City, State, and ZIP Code) <b>Monterey, California 93943-5000</b>		
8a NAME OF FUNDING / SPONSORING ORGANIZATION		8b OFFICE SYMBOL (if applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code)			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO	PROJECT NO	TASK NO
			WORK UNIT ACCESSION NO		
11 TITLE (Include Security Classification) <b>THE DUAL DECOMPOSITION METHOD AND ITS APPLICATION TO AN INTERDICTED NETWORK</b>					
12 PERSONAL AUTHOR(S) <b>GANNON, Timothy Paul</b>					
13a TYPE OF REPORT <b>Master's Thesis</b>		13b TIME COVERED FROM TO	14 DATE OF REPORT (Year, Month, Day) <b>1989 March</b>		15 PAGE COUNT <b>75</b>
16 SUPPLEMENTARY NOTATION <b>The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government</b>					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Network, Interdiction, Transportation, dual decomposition		
19 ABSTRACT (Continue on reverse if necessary and identify by block number) <p>This paper introduces the dual decomposition method for determining the distribution of an optimal objective function for a network problem. The objective function is to minimize the shortfall of demands to prioritized sinks for a four day period over a network that is subject to interdiction. The requirements of the model are that the upper and lower bounds on the capacities of the arcs and nodes of the network and the probabilities of interdiction are known. The dual decomposition method is an iterative approach to enumerating the possible instances of capacities in a capacitated network, based on the dual variables of the previous iteration. The purpose of the procedure is to determine the distribution of the shortfall of demands so that logistics planners can predict the performance of a supply distribution system over a short period of time.</p>					
20 DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION <b>Unclassified</b>		
22a NAME OF RESPONSIBLE INDIVIDUAL <b>Michael P. Bailey</b>			22b TELEPHONE (Include Area Code) <b>408-646-2085</b>		22c OFFICE SYMBOL <b>55Ba</b>

Approved for public release; distribution is unlimited

The Dual Decomposition Method and Its Application  
to an Interdicted Network

by

Timothy Paul Gannon  
Captain, United States Army  
B.S., United States Military Academy, 1979

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL  
March 1989

## ABSTRACT

This paper introduces the dual decomposition method for determining the distribution of an optimal objective function for a network problem. The objective function is to minimize the shortfall of demands to prioritized sinks for a four day period over a network that is subject to interdiction. The requirements of the model are that the upper and lower bounds on the capacities of the arcs and nodes of the network and the probabilities of interdiction are known. The dual decomposition method is an iterative approach to enumerating the possible instances of capacities in a capacitated network, based on the dual variables of the previous iteration. The purpose of the procedure is to determine the distribution of the shortfall of demands so that logistics planners can predict the performance of a supply distribution system over a short period of time.

5114377  
C.1

## THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

## TABLE OF CONTENTS

I.	INTRODUCTION . . . . .	1
	A. OBJECTIVE . . . . .	1
	B. GENERAL . . . . .	2
	C. LITERATURE REVIEW . . . . .	6
	D. ORGANIZATION . . . . .	11
II.	FORMULATION OF THE FOUR DAY LOGISTICS PROBLEM .	12
	A. INTRODUCTION . . . . .	12
	B. DESCRIPTION OF NETWORKS . . . . .	12
	C. MODEL DEFINITIONS . . . . .	15
	D. FORMULATION AS A LINEAR PROGRAM . . . . .	18
	E. NETWORK MODELLING TECHNIQUES . . . . .	21
III.	THE DUAL DECOMPOSITION METHOD . . . . .	23
	A. INTRODUCTION . . . . .	23
	B. APPLICATION OF THE METHODOLOGY . . . . .	23
	C. DUALITY AND THE MULTIDAY LOGISTICS PROBLEM	25
	D. THE DUAL DECOMPOSITION METHODOLOGY . . . .	26
	E. EXAMPLE PROBLEM . . . . .	30
	F. COMPARISON OF RESULTS OF DUAL DECOMPOSITION METHOD AND OTHER METHODOLOGIES . . . . .	38
IV.	COMPUTATIONAL EXPERIENCE . . . . .	40
	A. INTRODUCTION . . . . .	40

B.	RESULTS OF A FOUR DAY LOGISTICS PROBLEM DETERMINED BY THE DUAL DECOMPOSITION METHOD . . . . .	40
C.	EFFECTS OF VARYING THE VALUE OF THE THRESHOLD VALUE . . . . .	44
D.	DESCRIPTION OF THE COMPUTER PROGRAM . . . .	47
V.	SUMMARY AND FUTURE RESEARCH AREAS . . . . .	50
A.	SUMMARY . . . . .	50
B.	FUTURE RESEARCH AREAS . . . . .	51
APPENDIX	COMPUTER PROGRAM . . . . .	54
LIST OF REFERENCES	. . . . .	63
INITIAL DISTRIBUTION LIST	. . . . .	65

## LIST OF TABLES

1. Summary of Linear Program Variables . . . . .	18
2. Summary of Results of Example Problem by Different Methodologies . . . . .	38
3. Summary of Results of Four Day Logistics Problem by Different Methodologies . . . . .	46

## LIST OF FIGURES

1. Example Network . . . . .	31
2. First Branch Diagram of Example Problem . . . . .	33
3. Second Branch Diagram of Example Problem . . . . .	35
4. Final Branch Diagram of Example Problem . . . . .	36
5. Approximate Optimal Objective Function for Example Problem . . . . .	37
6. Approximate Optimal Objective Function for Minimum Cost Network Flow Problem (Threshold value = 7.0) . . . . .	42
7. Approximate Optimal Objective Function for Minimum Cost Network Flow Problem (Threshold value = 15.0) . . . . .	45

## I. INTRODUCTION

### A. OBJECTIVE

The objective of this paper is to provide a method for determining the approximate distribution of an optimal objective function of a linear program that minimizes the shortfall of supplies, over a period of several days, on a network that is interdictable. The supplies flow through arcs and nodes that have variable capacities to prioritized sinks. The procedure that is used to determine the distribution is the dual decomposition method. This procedure will be applied to an example network and the results will be analyzed. The result of the application of this method is that logistics and transportation planners can predict the performance of the supply distribution system over a short period of time.

The dual decomposition method is an iterative process for enumerating the possible flows through a network, based on the dual variables of the previous solution. This method is an attractive alternative in identifying the most likely patterns of performance of a network because complete enumeration is not required. The model is easy to use and understand and can be calibrated to meet the needs and capabilities of the planner.

This method has obvious applications in the military logistics planning field. The logistics planners can use the method as a tool for determining the stockage levels of depots, the allocation of rear area protection forces to key segments of the transportation system, and to forecast periods of critical shortages. The predictive capability of the dual decomposition method allows the military commander to make more informed decisions on tactical matters.

#### B. GENERAL

The focus of this study is a military, land transportation network that has a limited capacity and is interdictable. The dual decomposition solution method can be applied to tactical and strategic movement routes that are multimodal, however for the purposes of this study the networks analyzed will be ground, tactical movement routes. The ground transportation system, whether it is highway, rail, or inland waterway, carries the vast majority of the required tonnages to the forward combat units. Therefore, it is the most important system to the military transportation planner.

The transportation of supplies to military units involved in combat is a vital requirement of the United States Army's Air-Land Battle (ALB) fighting doctrine. The successful execution of this doctrine, as stated in the Army's Field

Manual 100-5, will depend on four basic tenets: initiative, agility, depth, and synchronization. [Ref. 1]

The intensity and speed of movements associated with the ALB doctrine will require the transportation movements planner and manager to be able to have fast, efficient tools to assist in the movement of supplies to support the combat units. The United States Army's Transportation Schools Field Circular 55-16, Movements Control Officer's Battle Support Guide, states that the planner must be able to maintain:

- Uninterrupted support in spite of unit relocations;
- Dispersion of supply activities;
- The movement of cargo despite enemy interdiction throughout the theater and the depth of the battlefield. [Ref. 2]

The requirements of the highly mobile, high-technology weapon systems will be enormous. The timely receipt of critical supplies will be vital to the successful execution of the ALB doctrine. The transportation community must be able to provide this uninterrupted support over networks that are subject to interdiction.

Interdiction may be the result of sabotage by enemy SPETNATZ troops, enemy airstrikes, enemy missile strikes, and environmental factors, such as the weather. A high priority target of the enemy's assets will be the lines-of-communications (LOCs) and logistics facilities, such as depots and transportation centers. The securing of these objectives

directly supports the Soviet concepts of surprise, mobility, deep penetration, and rapid exploitation. The Soviets are prepared to attack in considerable depth to destroy these targets and to harass the rear area operations [Ref. 3]. The transportation network may also be congested with the movements of friendly units and noncombatants which, in effect, interdict the network.

This problem is not new to transportation planners. Considerable effort has been devoted to analyzing the problem of transporting supplies over a network that may be interdicted. The next section will review work that has been done in this area.

Historical records indicate that military transportation planners have used extraordinary measures to ensure that critical supplies are delivered to units engaged in combat. For example, in World War II the Red Ball Express was developed to transport supplies to the combat units. The Red Ball Express was a term given to the military motor transport units that were centralized under a command with the mission of delivering critical supplies to the rapidly advancing combat units. The Red Ball Express was organized because the existing railways could not meet the demands generated by the armies' advance beyond the Rhine River. These units travelled on designated one-way roads so that there would be minimal interruption of supplies to the forward units. Despite its

short life, the Red Ball Express was an effective method of delivering priority supplies.

The Red Ball Express's accomplishments were impressive and widely acclaimed. During its eighty-one days of operation the transportation units carried an average of 5,088 tons per day. Despite the accomplishments of the Red Ball Express, the operation highlighted some serious deficiencies in the movement of cargo in a wartime theater. Centralized control of the movements was weak, the maintenance of the vehicles was poor, critical supplies, such as gasoline and tires, were consumed at an abnormally high pace, and it did not have sufficient cargo-handling capacity for long term operations. [Ref. 4]

The Red Ball Express's average daily tonnage during its operation may have been sufficient in World War II, however today's modern weaponry and tactical systems require substantially more support. For example, a mechanized infantry division is expected to consume 20,000 tons of supplies during a day of offensive operations.

It is necessary to capitalize on these lessons to prevent the lack of transportation support in future conflicts. In 1946, Secretary of War Patterson said that in future wars the most efficient means of transport will be required and that "...changes in favor of speed and flexibility will make what we now have seem primitive." [Ref. 5]

It is for this reason that military transportation planners must be able to analyze a transportation network in an effective and efficient manner so that the requirements are satisfied on time. By applying the dual decomposition method to a network the planner can determine the distribution of shortfall of supplies to the combat units. From this distribution the logistician and commander can analyze what effect this shortfall will have on combat operations, if any. This paper will illustrate one method that the planner can use to help ensure that the transportation system is receptive to the requirements of the modern battlefield.

### C. LITERATURE REVIEW

The problem of analyzing the effects of interdicting networks has been studied for several years. There have been several different approaches to the analysis. One general approach has been to use linear programming and network flow theory and another approach has been to use stochastic methods and simulations to describe the performance of a network. When the arcs have variable capacities the different methodologies rely on the enumeration of all possible routes through a network.

Ford and Fulkerson [Ref. 6] developed the maximum flow - minimum cut theorem. This theorem establishes that the maximum flow from a single source to a single sink is given

by the value of the minimum cut that separates these nodes. This underlying theory forms a basis for developing more sophisticated network flow algorithms.

Muster and McMasters [Ref. 7] developed an algorithm that determined which arcs in a network were the most susceptible to attack and how many of the available aircraft should be allocated to attacking a particular arc. The method used was a linear programming model that minimized the capacity of a cut set of a network subject to constraints on the number of available aircraft and the bounds on the capacity of the arcs of the network. The algorithm used a linear relationship between arc capacity and resource availability and was based on the maximum-flow minimum-cut algorithm developed by Ford and Fulkerson. Their method can only be used on planar networks that have a single source and single sink. Additionally, the effect on the flow through the network is for one point in time rather than over a several day period.

Preston and Howard [Ref. 8] developed a similar procedure to assign the optimum allocation of aircraft against a transportation network. Their method differed from Muster's and McMasters' in that they used an exponential relationship between arc capacity and resource availability in their algorithm. The purpose of the algorithm was to determine how to minimize the capacity of the network subject to aircraft availability. They determined the allocation of the aircraft

against the network so that the incremental increase in number of aircraft to a mission is exceeded by the benefit resulting from disrupting the flow through the network. A limitation of this model is that the cost function is a measurement of the use of an aircraft versus the benefit of interdicting a network. The cost function must be varied in accordance with tactical considerations and is difficult to establish. This algorithm also has the same limitations as the Muster and McMasters paper. The algorithm is based on the minimization of flow for one day and the network has a single source and a single sink and was deterministic.

Wollmer [Ref. 9] presented two algorithms for targeting strikes against a lines-of-communication network. His algorithms attempted to make the cost of achieving a circulation of flow between two points as large as possible over time. This was done by increasing the arc-cost functions and decreasing the arc capacities for a given period of time given the effect of targeting strikes. His first algorithm assumed the arc costs are linear functions of flow and his second one treated the arc costs as piecewise linear functions with one break point. The results of his algorithms enable the user to determine the effects of single and multiple strikes on the flow through a network for a point in time. Wollmer concluded that arcs that have a variable capacity based on the probability of interdiction can be replaced with

arcs that have capacities that are equal to the expected value of the capacity. The conclusion that arcs with variable capacity can be replaced with arcs that have capacities equal to their expected value is incorrect. The expected value of an arc's capacity is a central value and the flow through a network is dependent upon the extreme values of the arc's capacities. Wollmer's algorithms can be applied to planar networks with one source and sink.

Other authors used stochastic methods to analyze the effects of interdiction on networks. Four noteworthy papers are described below.

Fishman [Ref. 10] described a Monte Carlo sampling plan for estimating the distribution of maximum flow in a directed network whose arcs have random capacities. His procedure determined an estimate of the performance of a network as time evolves given the joint distribution of the capacities of the arcs in the network. The network can represent a multistate system whose capacities are subject to random deterioration. While his paper does not deal with the network being interdicted directly, it does provide an estimate of the probability of the maximum possible flow through a network and the variance of this estimate.

Bellovin [Ref. 11] analyzed a stochastic transportation network. He presented two related algorithms for analytically determining the value of the reliability of a network and the

expected quantity of unsatisfied demand. The networks that were analyzed had random demands and the arcs and supply nodes were subject to failure with known distributions. Bellovin's algorithm computed the shortage distribution of a stochastic, small to moderately-sized, transportation network. His algorithm determined an expected quantity of the shortfall of supply rather than a distribution of the shortfall.

Evans [Ref. 12] considered capacitated networks in which the branch capacity is an independent random variable with a known probability distribution. He investigated the computation of the probability distribution of the maximum flow in the network. His initial approach was to completely enumerate the capacity state vectors. The probability that the maximum flow has a specific value was found by summing the probabilities of all capacity vectors whose minimum cut set value equals the value of the maximum flow. Evans recognized that this approach produced computational difficulties for large networks. He then illustrated how a lattice structure can be used to reduce the computational requirements. The lattice is a set of cut sets of the network. Any two lattices that have comparable upper and lower bounds can be expressed as a single lattice. This approach may reduce some computations, but the number of cut sets grows exponentially as the network size increases.

Wollmer [Ref. 13] authored another paper on an interdiction model for sparsely traveled networks. He presented an algorithm for choosing locations to place forces in order to prevent the enemy from proceeding through the transportation network. His model calculated the probability of placing the force at particular arcs and nodes and the expected number of forces that should be placed on the network. His model used game theory as the approach of placing protection forces against an infiltrator. This model did not deal with the degradation of arc capacities because of enemy attacks but it is useful in the assignment of protection forces on a transportation network.

#### D. ORGANIZATION

Chapter II presents the formulation of the four day logistics problem. The dual decomposition method will be detailed in Chapter III. Computational experience will be discussed in Chapter IV. The conclusions of the research and recommendations for further research will be addressed in the final chapter, Chapter V.

## II. FORMULATION OF THE FOUR DAY LOGISTICS PROBLEM

### A. INTRODUCTION

This chapter begins with an overview of network preliminaries. The formulation of the four day logistics problem for a combat setting and presentation of the model will be presented in subsequent sections. This chapter will provide the necessary background for the development of the dual decomposition solution method which is presented in Chapter III.

### B. DESCRIPTION OF NETWORKS

A graph,  $G = (N,E)$ , is a set of elements,  $N$ , and a set of unordered pairs of elements from  $N$ ,  $E$ . The elements of the set  $N$  are referred to as vertices or nodes and the set  $E$  is the set of edges or arcs. The nodes are numbered by integers and are normally denoted by  $i$  and  $j$ . Nodes of a graph are numbered from 1 to  $n$ . Each edge is represented by  $(i,j)$ , where  $i$  is the tail node (where the edge originates from) and  $j$  is the head node.

A network is a graph with additional properties, parameters, or values associated with the graph's vertices and edges.

In a network representation of a transportation system, the nodes can represent supply depots, ports, transshipment points, critical road junctions, or a variety of other distinguishable features. The edges on the network represent the roads, rail lines, pipelines, or other avenues that supplies can travel over. Normally edges have labels that identify the edge's length, capacity, vulnerability, or any other characteristic or combination of characteristics. Nodes may also have properties and be labelled.

The movement of supplies over a transportation network is analogous to a liquid moving through a pipe. The network, like a pipe, has a finite capacity. This capacity can be represented by such measures as vehicles per day, tons per mile, or any other relationship which describes the limits of the arc's ability to carry goods. A network which has capacitated arcs is called a capacitated network. The capacity of an arc may depend on the width of the road, the road's surface, weather, or the configuration and capacity of the vehicles that travel over the arc.

Nodes may also have capacities. For example, a supply depot has a finite storage capacity. Again, these capacities can be described in much the same manner as an arc's capacities.

Other characteristics of a network include sources and sinks. A source node is a node that originates flow and a

sink node is any node where flow terminates. In the transportation network, a source node may be a supply depot or port where cargo enters the network. A sink node may be a unit who consumes the supplies or an intermediate supply depot that stores supplies. Some algorithms only consider the network to have a single source and sink node. If the network being modelled has multiple sources or sinks and the objective function is the sum of flows, a super source and super sink can be constructed. A super source and sink can be constructed by adding dummy edges from the sink and source nodes to the respective super nodes. These dummy edges have infinite capacity.

The underlying graph of the transportation network is assumed to be a sparse, planar, directed graph. A sparse graph is a graph where the cardinality of the edges,  $|E|$ , is proportional to the cardinality of the vertices,  $|N|$ . A graph is planar if it can be drawn in the plane with no arcs crossing. A graph is directed if there is only one orientation of the arc. That is, commodity flows can only move in one direction along the arc.

The dual decomposition method does not require the graph be sparse or planar, however these attributes of the graph tell us about the performance of the method.

The performance of a network can be measured and optimized by the application of linear programming methods. Some

reasonable objective functions of a linear program are to maximize flow through a network, to minimize the cost associated with the movement of supplies through a network, or to minimize the time required to move supplies through a network. The objective function that is considered in this paper is to minimize the summed prioritized shortfall of demand at the sinks.

### C. MODEL DEFINITIONS

The following section will define the terms associated with the model defined in the next section.

The supplies that travel through an arc  $(i,j)$  during a time period,  $k$ , are indicated by  $f_{i,j,k}$ . For the purposes of this paper, the supplies are the requirements of the combat units. These units require subsistence, petroleum and lubricants, barrier material, ammunition, repair parts, and major end items such as trucks and tanks.

If a demand is not satisfied at a sink a shortfall will occur. The shortfall of supplies at a sink  $j$  during a time period,  $k$ , is represented by  $sf_{j,k}$ . The supplies that do get to a sink plus the shortfall equal the demand.

The maximum capacity of an arc that has not been interdicted during a specific time period,  $k$ , is represented by  $u^1_{i,j,k}$ . Each arc is vulnerable to interdiction by the enemy. The vulnerability of the arc can be expressed as a

probability that the arc will be interdicted and therefore, have a reduced capacity,  $u_{i,j,k}^2$ . The probability that arc  $(i,j)$  is interdicted on day  $k$  is represented by  $p_{i,j,k}$ . The probability an arc is not interdicted is  $1 - p_{i,j,k}$ . The capacity of an arc depends on the width of the road, the road's surface, weather, the capacity of vehicles using the arc, and whether the arc has been interdicted.

The demand at a sink node,  $j$ , for a time period,  $k$ , is indicated by  $D_{j,k}$ . This demand, measured in short tons and gallons, can be determined by summing the requirements for all commodities for that particular node or by standard planning factors. Standard planning factors are guidelines that provide the planner some rules of thumb on the expected consumption rates of different commodities. They are based on historical usage rates of the different supply commodities. They can be used to estimate the requirements of a particular type of unit during the planning phase or when the requirements are not explicitly known. These factors are available in U.S. Army Field Manual 101-10-1, Staff Officers' Field Manual, Organizational, Technical, and Logistic Data, and Reference Book 101-999, Staff Officers' Handbook. It will be assumed that the supplies entering the network are not constrained by availability.

The inventory of supplies that is held at a depot node,  $j$ , between time periods  $k-1$  and  $k$  is denoted by  $I_{j,k}$ . The

capacity of a node,  $j$ , for a time period,  $k$ , is denoted by  $c_{j,k}$ . The capacity of a node can be restricted by the size of the facility, availability of material handling equipment, and a variety of other factors. The sum of the flows into a node minus the sum of flows out must be less than the capacity of the node during time period  $k$ . If the node is at its maximum capacity and is not being interdicted the capacity is denoted by  $c^1_{j,k}$ . The notation for a depot that is being interdicted is indicated by  $c^2_{j,k}$ .

A weighting factor for a particular sink node and time period is represented by  $w_{j,k}$ . The weighting factor allows the planner to prioritize the shipment of cargo to a particular sink in accordance with the priority of support determined by the commander. A large weighting factor implies the node has priority and shipments should arrive in a timely manner. Each sink has a different priority, weighting factor, that corresponds to the priority established by the commander of the forces receiving the supplies.

The set of nodes that represent the sinks is denoted by  $T$ . The set  $D$  is the set of all depot nodes. The problem will be solved for  $K$  days.

The objective function of the model is to minimize the summed, weighted shortfall of the demand for each sink and time period.

The variables used in the model are summarized below in Table 1.

TABLE 1 SUMMARY OF LINEAR PROGRAM VARIABLES

$f_{i,j,k}$	- flow from arc $i$ to arc $j$ on day $k$
$sf_{j,k}$	- shortfall of supplies at node $j$ during day $k$
$u^1_{i,j,k}$	- the maximum capacity of an uninterdicted arc $(i,j)$ during day $k$
$u^2_{i,j,k}$	- the capacity of an interdicted arc $(i,j)$ during day $k$
$P_{i,j,k}$	- the probability that an arc $(i,j)$ is interdicted on day $k$
$D_{j,k}$	- the demand at node $j$ during day $k$
$I_{j,k}$	- the inventory at node $j$ between day $k-1$ and $k$
$c^1_{j,k}$	- the uninterdicted capacity of node $j$ during day $k$
$c^2_{j,k}$	- the interdicted capacity of node $j$ during day $k$
$w_{j,k}$	- priority of sink $j$ on day $k$

#### D. FORMULATION AS A LINEAR PROGRAM

The problem for the logistician and the movements planner is to determine the distribution of the shortfall of supplies so that some insight can be gained on the performance of the supply distribution system over a period of time. The distribution of the optimal shortfall involves minimizing the shortfall of supplies to the prioritized sinks for each day. The minimization is subject to the balance of flow in and out of the depot nodes and the conservation of flow at non-depot

nodes. Additionally, constraints on the capacities of the arcs and depot nodes exist.

The problem can be written mathematically as:

$$\text{Minimize } \sum_{j \in \text{sinks}} \sum_{k=1,2,\dots,K} w_{jk} (sf_{jk}) \quad 1.1$$

Subject to

$$\sum_{i \in N} f_{i,j,k} + sf_{j,k} = D_{j,k} \quad \text{for all } j \in T \quad 1.2$$

$$k = 1, 2, \dots, K$$

$$- \sum_{i \in N} f_{i,j,k} + I_{j,k-1} \geq 0 \quad \text{for all } j \in D \quad 1.3$$

$$k = 1, 2, \dots, K$$

$$\sum_{i \in N} f_{i,j,k-1} + I_{j,k-1} - \sum_{i \in N} f_{i,j,k} - I_{j,k} \geq 0 \quad 1.4$$

$$\text{for all } j \in D$$

$$k = 2, 3, \dots, K$$

$$\sum_{i \in N} f_{i,j,k} - \sum_{i \in N} f_{j,i,k} \geq 0 \quad 1.5$$

$$\text{for all } j \in N - \{D \cup T\}$$

$$\text{for } k = 1, 2, \dots, K$$

$$0 \leq f_{i,j,k} \leq u_{i,j,k} \quad \text{for all } i, j \in N \quad 1.6$$

$$k = 1, 2, \dots, K$$

$$0 \leq I_{j,k} \leq c_{j,k} \quad \text{for all } j \in D \quad 1.7$$

$$k = 1, 2, \dots, K$$

Equation 1.1 is the objective function. The objective is to minimize the shortfall of supplies at sinks according to a certain priority or weighting factor for four time periods.

Equation 1.2 is the constraint determining shortfall from flows into the sink node. It states that the flow of supplies into a sink plus the shortfall equals the demand.

Equation 1.3 states that the flow of supplies out of a depot cannot be greater than the inventory of the depot. This equation, and Equation 1.4, imply that supplies cannot move directly through a depot node in one time period. These equations ensure a proper time sequencing of the flow of supplies through the network. A requirement of this model is that depot nodes must not be greater than one time period away from each other. The model is not limited by this requirement, however. The time period can be defined in whatever time increments that are suitable to the user. For this paper the time periods will be in terms of days.

Equation 1.4 states that the flow of supplies into a depot plus the previous day's inventory equals the flow out of a depot plus the ending inventory for that day.

Equation 1.5 is a constraint on the non-depots. It states the flow of supplies into a node during a time period must equal the flow of supplies out of a node during the same time period. This ensures that supplies are not stored at non-depot nodes.

Equations 1.6 and 1.7 are restrictions on the arcs and depots, respectively. The flow of supplies through an arc and node must be less than its maximum capacity but larger

than zero. The constraint on the lower bound of the capacities ensures that arcs are one directional and that depots can only ship supplies that are on hand.

#### E. NETWORK MODELLING TECHNIQUES

The linear programming problem described in the last section can be transformed to a network flow problem. This transformation enables the problem to be solved using more efficient and faster network solvers such as GNET [Ref. 14]. A network flow problem also produces solutions that are integer. Another benefit of the transformation to a network structure is that the physical characteristics of the problem are better represented by a network flow problem and they are more readily accepted by nonanalysts [Ref. 14: p. 2].

Some modelling techniques are required to transform the problem from a linear programming model to a network flow model. Artificial arcs are used to represent the flow of the inventory of supplies stored at depot nodes from one day to another. Artificial arcs are also required to maintain the conservation of supply and demand. In a network model the tonnage demanded must equal the tonnage supplied at the supply node. In the linear programming model described in the last section, a shortfall can exist at a sink. The objective function value is the amount of the shortfall times a priority factor. Since the objective of the network model is the same,

an artificial arc is used to capture the shortfall. In a minimum cost network flow problem the shortfall can be determined by having costs associated only with the arcs that carry the shortfall.

Another technique that is required for the transformation to a network flow model is the addition of another arc and node for every depot node. The depot node in the linear programming model has capacity constraints. The network model also requires constraints on the capacity of the depot inventory but only arcs in the network formulation can be constrained. It is necessary to construct an arc that represents the capacity of the depot node and another node to link the arc to the remainder of the network. This arc does not have the same physical interpretation as the other arcs in the network, but it does allow the capacity of the depot node to be constrained.

The next chapter discusses the dual decomposition method and its application to a network that can be interdicted.

### III. THE DUAL DECOMPOSITION METHOD

#### A. INTRODUCTION

This chapter gives the conditions under which the dual decomposition method can be applied to a network flow problem and the steps of the dual decomposition method. Then the dual decomposition method is applied to a simple example to illustrate how the methodology works.

#### B. APPLICATION OF METHODOLOGY

The dual decomposition method can be applied to a network where the arcs have known higher and lower capacities. The higher capacity is the capacity of the arc when it is not interdicted and the lower capacity is the capacity of the arc when it has been interdicted. Generally, the capacities are expressed in terms of rates of flow, such as tons per day or gallons per hour. The values of these capacities can be determined from historical data, subject matter experts, and some capacity values can be found in the DAMSEL network data base [Ref. 15]. This data base has unrestricted capacity values for the majority of the transportation networks in Europe. A lower capacity of an arc may be zero but it has been determined from studies of network capacities in wartime that an arc in the network normally has some positive value

because the damage done by interdiction can be repaired in a short amount of time [Ref. 16].

The probability of interdiction must also be known. Again, the probability of interdiction can be determined in the same manner and from the same sources as the arc capacities. The probability of interdiction normally depends on how the enemy perceives the tactical or strategic value of the arc.

The dual decomposition method determines the distribution of the optimal objective function of the minimum cost network flow problem without having to enumerate all possible combinations of arc capacities. This is especially critical given that each arc has two possible arc capacities and there may be hundreds of arcs in a network. If there are  $n$  arcs in a network, the determination of the true distribution of the optimal objective function by full enumeration will require solving  $2^n$  minimum cost network flow problems. It will be illustrated that the dual decomposition method can determine an approximate optimal objective function with substantially fewer problems being solved.

In deterministic optimization, the expected value of the capacity of each arc is used. A single minimum cost flow problem is then solved. The dual decomposition method also precludes having to use an expected value for the arc's capacity. The use of expected values for arc capacities does

not provide an expected value of a minimum cost network flow problem because the minimum cost flow depends on extreme values of the arc capacities rather than central values. This will be illustrated during the example problem later in the chapter.

### C. DUALITY AND THE MULTIDAY LOGISTICS PROBLEM

The multiday logistics problem presented in Chapter II may be rewritten as:

$$\text{Minimize } c^T f = Z \quad 3.1$$

$$\text{Subject to } Af = b$$

$$If \leq u$$

$$f \geq 0$$

The vector of flows is represented by  $f$ ,  $c$  is the cost vector, and  $A$  is a  $K(|N| + |D|)$  by  $K|E|$  matrix encapsulating the constraint equations 1.1 through 1.5.  $I$  is a  $K(|D| + |E|)$  dimensional identity matrix. The vector of arc capacities is denoted by  $u$ .

The dual to the above linear program is given by:

$$\text{Maximize } b^T v + u^T w = Z \quad 3.2$$

$$\text{Subject to } v^T A + w^T I \geq c$$

$$w \geq 0$$

The vectors  $v$  and  $w$  are the dual variables of the problem. A basic result in duality theory is that, at optimality, the objective function values of these two problems are equal.

Let  $i \in E$ , where  $E$  is the set of edges, then  $dZ/du_i = w_i$ , as evidenced by the objective function of 3.2. Hence, for a given set of capacities the arc with the largest optimal dual value can be seen to be the arc which is the most crucial in determining the value of the objective function,  $Z$ .

The network solver GNET was used to solve instances of the minimum cost network flow problem represented by the linear program given at 3.1. The GNET code was modified in order to determine the vector  $w$  post optimally. This modification does not significantly impact the performance of the GNET code.

#### D. THE DUAL DECOMPOSITION METHODOLOGY

As has been mentioned, we are interested in developing an iterative approach to the decomposition of the stochastic minimum cost network flow problem. The methodology involves starting with all the arcs "free", that is, each arc may be at its upper or lower capacity, the capacity being still random. The set of free arcs will be referred to as the set  $F$ .

The starting problem is called "problem 0." At the outset,  $F_0 = E$ . The subscript indicates the problem number. Two minimum cost network flow problems are solved, subproblem A has all arc capacities set to their lower bounds, while subproblem B has all arc capacities set at their upper bounds. After solving these two problems using the modified version of GNET, the dual vectors  $w_A$  and  $w_B$ , as well as the objective

function values  $Z_A$  and  $Z_B$ , are determined. Recall from the discussion of the dual of the multiday logistics problem that for each arc  $i$ ,  $w_{A,i}$  is the change in the objective function value for one unit of change in the capacity of arc  $i$  given that all other arcs stay at their lower capacities. Thus the arc  $i^*$  which maximizes  $w_{A,i}$  is the arc of greatest influence in subproblem A, while the arc  $j^*$  which maximizes  $w_{B,j}$  is the arc of greatest influence in subproblem B. Among  $i^*$  and  $j^*$ , arc  $k^*$  is chosen which maximizes  $\{w_{A,i} : i \in E\} \cup \{w_{B,j} : j \in E\}$ . Arc  $k^*$  will be called the "pivot arc" for problem 0. If we condition on the capacity of arc  $k^*$ , we will gain significant insight into the value of the objective function.

Let problem 01 have the capacity of arc  $k^*$  set to its upper capacity, while problem 00 has arc  $k^*$  set to its lower capacity. The free arcs for both problems are given by  $F_{01} = F_{00} = E - \{k^*\}$ . We assign probabilities of occurrence to each of these new problems.  $P_{01}$  is the probability that arc  $k^*$  is set to its upper capacity. In the original problem formulation, this corresponds to arc  $k^*$  being unharrassed.  $P_{00}$  is given by the probability that arc  $k^*$  is interdicted, implying its lower capacity prevails.

The next step in the process is finding values of  $Z_1$  and  $Z_2$  for problems 01 and 00. For problem 00, both subproblems have the capacity of arc  $k^*$  (all the arcs in  $E - F$ ) set to its upper capacity. In problem 01 subproblem B, the free arcs are

set to their upper capacity to find the new  $Z_2$  value. In problem 01 subproblem A, the free arcs are set to their lower capacity. Note that, the new  $Z_2$  value equals the  $Z_2$  value of problem 0 because the arc capacities for all arcs are the same.

In problem 00 subproblem B, the free arcs are set to their upper capacity to find the new  $Z_2$  value, for subproblem A of problem 00, the  $Z_1$  value is found by setting the free arcs to their lower capacity. Every arc in  $E - F$ , that is arc  $k^*$  in this case, is set to its lower capacity. The new  $Z_1$  value equals the  $Z_1$  value of problem 0 because, again, the arc capacities are identical. Thus in the first pair of problems, we have only needed to solve two new subproblems.

In problem 01, the vectors of dual variables for both subproblems are analyzed to find the new arc  $k^*$  which maximizes  $\{w_{A,i} : i \in E\} \cup \{w_{B,j} : j \in E\}$ . This gives the pivot arc for problem 01.

In problem 00, the same procedure is used. The vector of dual variables is analyzed to find the arc  $k^*$  which maximizes  $\{w_{A,i} : i \in E\} \cup \{w_{B,j} : j \in E\}$  for problem 00. This yields the pivot arc for problem 00.

We continue by generating problems 001 and 000 from problem 00 and problems 010 and 011 from problem 01.

The next step in the dual decomposition process is to check the termination criteria for both problems 00 and 01.

A problem can be terminated if it can be determined that further decomposition of the problem will not produce significant improvements in the approximation of the distribution of the optimal objective function. The termination criteria measures the absolute difference between the problem's  $Z_1$  and  $Z_2$  values multiplied by the probability that the nonfree arcs realized their prescribed values for that problem. The equation for the termination check is provided at Equation 3.3.

$$|Z_1 - Z_2| * p \leq \text{threshold value} \quad 3.3$$

Where  $p = P_{00}$  for problem 00 and  $p = P_{01}$  for problem 01.

The threshold value is determined by the user of the methodology. The threshold value must be small enough so that the approximate distribution of the optimal objective function that results from the application of the methodology adequately represents the actual optimal objective function.

The iterative process of constructing subproblems A and B yielding  $Z_1$  and  $Z_2$ , respectively, checking for termination, finding the pivot arc and probability, and creating the two daughter problems continues until all problems terminate.

If the termination criteria is met, the values of  $Z_1$ ,  $Z_2$ , and  $p$  are added to a SOLN set. The SOLN set is the set of  $Z_1$ ,  $Z_2$ , and  $p$  values that will determine the approximate distribution of the optimal objective function.

The probability density function of the minimum cost flow for the network may be constructed by plotting the values of  $Z_1$ ,  $Z_2$ , and  $p$  from the SOLN set. For each terminated problem we assume that the objective function value for a terminated problem is uniformly distributed between  $Z_1$  and  $Z_2$  with probability  $p$ . The result of the combination of the individual  $Z_1$ ,  $Z_2$ , and  $p$  values is the approximate distribution of the optimal objective function.

#### E. EXAMPLE PROBLEM

This section contains an example problem that illustrates the dual decomposition method. For the purpose of illustrating the methodology, a different, simpler model than the four day logistics problem described in Chapter II will be used. This linear program's objective is to maximize the flow through the network. The only constraints are the balance of flow constraints through the nodes and the capacity constraints of the arcs. The model is described mathematically below.

maximize  $f$

such that

$$\sum_{j \in N} x_{i,j} - \sum_{m \in N} x_{j,m} = \begin{cases} f & \text{if } i = 1 \\ 0 & \text{if } i \neq 1 \text{ or } T \\ -f & \text{if } i = T \end{cases}$$

$$x_{i,j} \leq c_{i,j}$$

$$x_{i,j} \geq 0$$

The flow through the network is represented by  $f$ , the flow through an arc  $(i,j)$  is represented by  $x_{i,j}$ , and the upper capacity of arc  $(i,j)$  is represented by  $c_{i,j}$ .

The example is a simple network with four arcs and four nodes. The example network is represented at Figure 1.

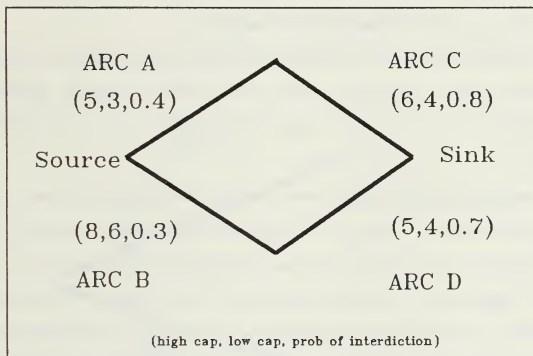


Figure 1. Example Network

The threshold value of the termination criteria will arbitrarily be set to 0.3 for this example.

The first step in the method is to solve the problem when the arcs are at their lower capacities and find the objective

value,  $Z_1$ . The problem is then solved at the upper capacities to find  $Z_2$ . When all arcs are at their lower capacities, the flow through the network,  $Z_1$ , is 7 units because the flow is restricted to the maximum capacity of Arcs A and D. The value of  $Z_2$  is 10 because Arcs A and D restrict the flow to 5 units in each branch of the network. The probability,  $p$ , is 1 since there is no pivot for the initial problem.

The termination criteria is checked and since the value is above 0.3 the decomposition begins.

The next step is to examine the dual variables of the two solutions and determine which arc has the most negative reduced cost. The arc with greatest impact on  $Z_1$  and  $Z_2$  values is Arc A.

The decomposition process begins with the problem being pivoted on Arc A. Arc A is fixed at its lower capacity and the problem is solved with the remaining free arcs at their lower capacities and then again at their upper capacities. The value of the objective function for the arcs at the lower capacity is the same as  $Z_1$  of the previous problem. The value of  $Z_2$  for this branch of the problem is 8.

Arc A is then fixed at its upper capacity and the problem is solved with the remaining free arcs at their upper and lower capacities. The value of  $Z_2$  in this case is the same as  $Z_2$  of the previous iteration. The value of  $Z_1$  for this branch of the decomposition is 8 (see Figure 2).

The termination criterion for both problem sets is checked next. The value of the termination criterion for the problem set with Arc A set its upper capacity is  $|8 - 10| * 0.6 = 1.2$ . The value of the termination criterion for the problem set with Arc A set at its lower capacity is 0.4. Since both problem set's solutions do not meet the termination criterion, the decomposition continues on both branches of the problem.

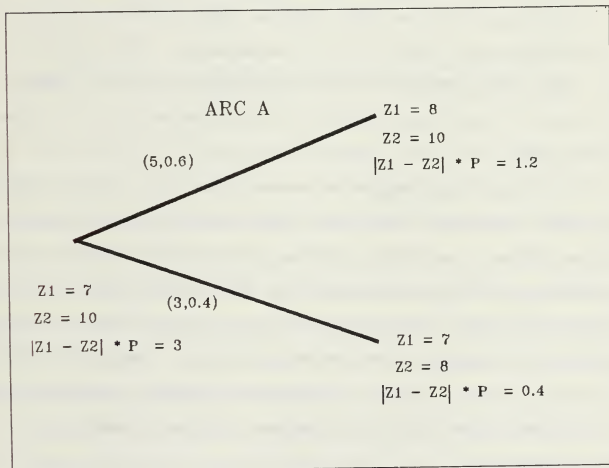


Figure 2. First Branch Diagram of Example Problem

The dual variables for the problem pair that had Arc A set to its upper limit indicated that the decomposition should continue with Arc C being fixed. The decomposition generates two additional problem pairs. The first problem pair is with the upper capacity of Arc C fixed and the values of  $Z_1$  and  $Z_2$  determined for the new branch. As in the first decomposition, the value of  $Z_2$  for this branch of the solution equals the value of  $Z_2$  from the previous solution, which was 10. The new value of  $Z_1$  for this problem set is 9.

The second problem pair has the lower capacity of Arc C fixed and the new value of  $Z_2$  is determined. The value of  $Z_1$  for this branch equals  $Z_1$  of the previous solution. The new  $Z_2$  value is 8 and the value of  $Z_1$  from the previous solution was 8 also.

The termination criteria values for both problem pairs are then calculated. The first problem pair had a termination criterion value of 0.12 and the second problem pair had a value of 0.0. Since both of these values are less than the threshold value, the decomposition on this branch of the problem is stopped. The values of  $Z_1$ ,  $Z_2$ , and  $p$  for both problems in this pair are added to the SOLN set. Figure 3 is a branch diagram of the problem at this point in the decomposition.

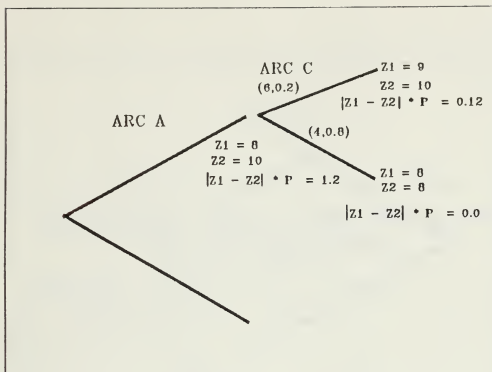


Figure 3. Second Branch Diagram of Example Problem

The analysis of the dual variables for the problem pair that had Arc A set to its lower capacity determined that Arc D should be pivoted on. The decomposition of the network continues at this branch with Arc D being set to its upper and lower capacity and two new problem pairs being generated. These two new problem pairs are solved and the termination criteria checked. If the termination criteria are not met the decomposition process continues.

Since both branches of the problem met the termination criteria the decomposition process ends. A branch diagram of the final structure of the problem, with all remaining values provided, is at Figure 4.

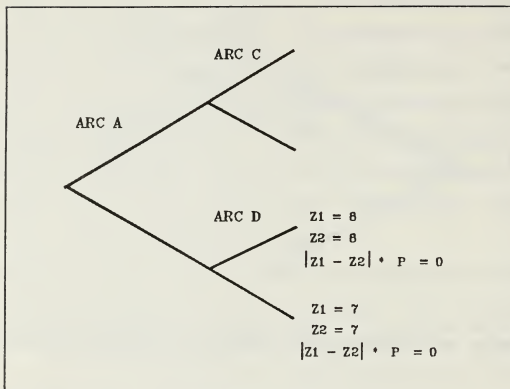


Figure 4. Final Branch Diagram of Example Problem

The distribution of the optimal objective function is then determined. The values in the SOLN set are plotted on an X-Y axis. The X axis is the values of the network flows,  $Z_1$  and  $Z_2$ , and the Y axis is the conditional probability. The  $Z_1$  and  $Z_2$  values determine the upper and lower limits on the network flow for that particular case. The probability density function of the network flow for the example problem is provided at Figure 5.

### Approximate Optimal Objective Function for Example Problem

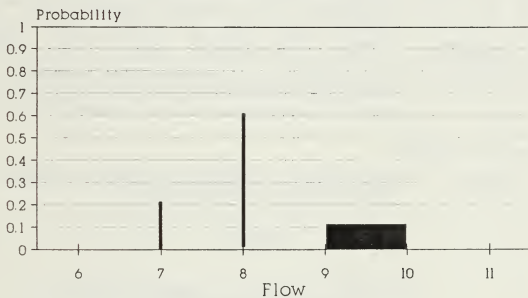


Figure 5. Approximate Optimal Objective Function  
for Example Problem

F. COMPARISON OF RESULTS OF DUAL DECOMPOSITION METHOD AND OTHER METHODOLOGIES.

Determining the exact distribution of the flow for the example network would require the complete enumeration of all possible flow combinations, solving  $2^4 = 16$  minimum cost network flow problems. The dual decomposition method only required solving 6 minimum cost network flow problems.

The solution of the network flow problem by using expected values for arc capacities only requires solving one network flow problem, however the result is not an accurate representation of the actual flow through the network.

Table 2 provides a summary of the comparison of the different results.

TABLE 2. COMPARISON OF RESULTS OF EXAMPLE PROBLEM BY DIFFERENT METHODOLOGIES

TABLE 2		
COMPARISON OF RESULTS OF EXAMPLE PROBLEM BY DIFFERENT METHODOLOGIES		
Type of Methodology	Flow	Problems Required
Full enumeration	8.02 (actual)	16
Dual Decomposition	7.9	6
Expected Value	8.7	1

The result of the problem by full enumeration is the distribution of the actual flow through the network, however

it requires solving 10 additional problems. The dual decomposition method provides a suitable approximation to the optimal objective function and only requires solving 6 problems. The solution by the expected value procedure is an overestimation by 8.7% of the true flow and does not provide an accurate representation of the flow through the network.

The variance of the full enumeration solution is 0.807. Therefore, the solution determined by the expected value method is not within one standard deviation of the actual solution. The solution determined by the dual decomposition method is well within one standard deviation.

#### IV. COMPUTATIONAL EXPERIENCE

##### A. INTRODUCTION

The first section of this chapter presents the results of a four day logistics problem when the dual decomposition method is applied to a network representation of the problem. The second section examines the effects on the approximate distribution of the optimal objective function when the threshold value of the termination criterion is varied. The third section presents the structure of a computer program that uses the dual decomposition method to determine the approximate optimal objective function for a minimum cost network flow problem.

##### B. RESULTS OF A FOUR DAY LOGISTICS PROBLEM DETERMINED BY THE DUAL DECOMPOSITION METHOD

A logistics scenario was developed to determine the approximate shortfall of supplies to two combat units. The linear program that represented the scenario was transformed into a minimum cost network flow model by using the techniques described in Chapter II.

The network representation of the logistics problem consisted of 57 arcs and 48 nodes. The objective of the problem was to minimize the shortfall of demands to two combat units over a four day period. Each unit had a priority of

support that varied with each day. Each arc had a higher capacity, a lower capacity, and a probability of interdiction. The network had 12 depots, 12 non-depots, 8 sinks, and one source. The network represented a fictitious highway network with capacities, probabilities, priorities, and demands established arbitrarily. The two combat units had a combined demand of 3125 units of prioritized supplies during the four day period.

The dual decomposition method was used to determine the approximate optimal objective function of the minimum cost network flow. The initial threshold value for the termination criterion was 7.0. The determination of the approximate distribution of the shortfall of supplies required solving 64 minimum cost network flow problems. After the termination criterion had been met for each pair of problems, the approximate distribution of the optimal objective function of the problem was constructed. The probability distribution function of the optimal objective function is in Figure 6.

It was determined from the probability distribution function that the prioritized shortfall of supplies to the two combat units for the four day period would have extreme values of 1350 and 1730 units of prioritized supplies. The mode of the approximate distribution was at 1550 units of prioritized supply. This is the most probable value of the

**Approximate Optimal Objective Function  
for a Minimum Cost Network Flow Problem  
(Threshold Value = 7.0)**

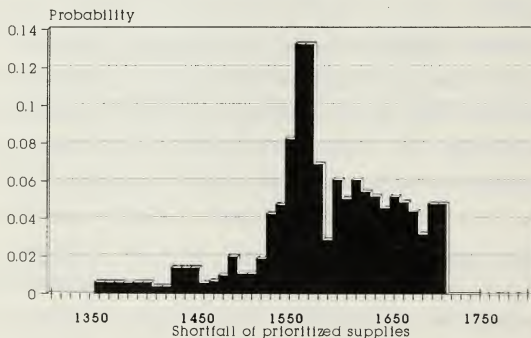


Figure 6. Approximate Optimal Objective Function for a Minimum Cost Network Flow Problem (Threshold Value = 7.0)

shortfall of prioritized demand. There are several "spikes" in the distribution, or local modes, between 1600 and 1680 units. The logistics planner can approximate that the shortfall of demand will be between 1550 and 1650 units in the shortfall of supplies 80% of the time.

Since the shortfall of supplies would be approximately 50% of the requirements, the logistics planner could recommend to the combat units' commander alternative methods of distributing the supplies and possibly how the shortfall could be avoided by increasing the stockage levels of supplies at the forward depots. The logistics planner could also recommend that certain segments, arcs, in the transportation network receive additional protection from interdiction thereby decreasing the probability of interdiction.

If the problem described above had been solved by complete enumeration of the arc capacities, the actual shortfall of supplies could be determined. However, this would require  $2^{57}$ , or approximately  $1.4 \times 10^{17}$ , minimum cost network flow calculations. The number of solutions required by the dual decomposition method is  $2^{-51} = 4.44 \times 10^{-16}$  of the total number of solutions required by full enumeration.

The logistics planner normally only requires an approximate value of the shortfall of supplies. The dual decomposition method can accomplish this approximation by

solving a very small fraction of the calculations that are required by the full enumeration procedure.

### C. EFFECTS OF VARYING THE VALUE OF THE THRESHOLD VALUE

If the threshold value of the termination criterion is adjusted to 15.0, only 32 minimum cost network flow problems must be calculated. This is a 50% reduction in the number of calculations required when the threshold value is 7.0.

All branches of the final decomposition tree were the same length. This will not normally be the case because the branch with a low probability will satisfy the threshold criterion earlier than the branch with a higher probability. In the four day logistics problem the probability of interdiction for each arc was normally 0.5 or 0.4. The probabilities for each branch in the tree were similar so the threshold criterion was satisfied at the same stage in the partial enumeration.

The approximate distribution of the optimal objective function for the minimum cost network flow with a threshold value of 15.0 is at Figure 7.

This distribution also has a mode at 1550 units of prioritized supplies but has a relative mode at 1500 units. The approximate distribution with the threshold value set to 15.0 is smoother than when it is set to 7.0. The logistics planner can approximate that 80% of the shortfall of demand will occur between 1490 and 1650 units of prioritized

**Approximate Optimal Objective Function  
for a Minimum Cost Network Flow Problem  
(Threshold Value = 15.0)**



Figure 7. Approximate Optimal Objective Function for a Minimum Cost Network Flow Problem (Threshold Value = 15.0)

supplies. This 80% confidence interval for the approximate distribution is wider than when the threshold value is 7.0 but it still can be used to determine an approximate shortfall of supplies to the combat units.

The following table illustrates the results of the four day logistics problem when it is solved using the dual decomposition method and when expected values are used for arc capacities.

TABLE 3. COMPARISONS OF RESULTS OF FOUR DAY LOGISTICS PROBLEM SOLVED BY DIFFERENT METHODOLOGIES

TABLE 3			
COMPARISON OF RESULTS OF FOUR DAY LOGISTICS PROBLEM SOLVED BY DIFFERENT METHODOLOGIES			
	Dual Decomposition Method Threshold Value		Arc Expected Value
	7.0	15.0	
Mode	1550	1550	-
Expected Value	1566	1570	1430
Std Deviation	140	150	-

The results illustrate that the solution of the method that uses expected values for arc capacities overestimates the capabilities of the network. The solution for this method has a lower value for the shortfall of prioritized supplies

than the solutions that used the dual decomposition method. This lower value for the shortfall of supplies gives an inaccurate and more optimistic prediction on the network's capabilities.

#### D. DESCRIPTION OF THE COMPUTER PROGRAM

The steps in a computer program for solving the multiple day logistics problem are provided below. The algorithm is written in pseudo-code so that it can be implemented in any programming language.

Read in the network characteristics

Construct the multiple day network flow representation of the problem

Construct the head, tail, capacity, cost, and probability arrays

Solve for  $Z_1$

Solve for  $Z_2$

IF(  $|Z_1 - Z_2| * P \leq$  threshold value) then

Record  $Z_1$ ,  $Z_2$ , and p values

STOP

ELSE

100

Choose arc i to be pivoted on

Put a new problem on the queue with arc i set to  
its upper capacity

Put a new problem on the queue with arc i set to  
its lower capacity

```

Take a problem off the queue
    Set all free arcs to upper capacity
Call GNET
    Record primal and dual variables
    Set all free arcs to lower capacity
    Call GNET
        Record primal and dual variables
IF(  $|Z_1 - Z_2| * P \leq \text{threshold value}$  ) then
    Record  $Z_1$ ,  $Z_2$ , and  $p$  values
IF( the queue is empty ) then
    STOP
ELSE
    GO TO 100

```

The queue that contains the problems to be solved is a first-in-first-out queue. The array that represents the problem to be put on the queue has values of  $Z_1$  or  $Z_2$  and  $p$  from the parent problem pair. If the problem being put on the queue is going to determine a new minimum cost network flow for the arc set to its lower capacity, the  $Z_2$  value is recorded. This prevents having to solve for the  $Z_2$  value, for that problem pair, since it will be the same as the last problem pair. The opposite holds if the arc is set to its upper capacity. The  $Z_1$  value is recorded since it is the same

as the previous problem pair. The conditional probability for the previous problem pair is also recorded since it will be used to determine the new conditional probability. The array that is stored on the queue also has a dimension that contains a cell for every arc. If the arc is not free the corresponding cell for that arc is marked so that when the problem is taken off the queue, the arcs that are not free can be set to their upper or lower capacity. The arcs will be set to their upper or lower capacity depending on how they are marked in the cell.

A FORTRAN program that uses the dual decomposition method to determine an approximate distribution of the optimal objective function for a minimum cost network flow problem is provided at Appendix A. This program uses the network solver GNET to solve the minimum cost network flow problem. The program also constructs a multiday network representation of the problem given the road network and depots.

The next chapter provides conclusions on the use of this methodology on network interdiction models and then provides recommendations on areas of further research that should be explored.

## V. SUMMARY AND FUTURE RESEARCH AREAS

### A. SUMMARY

The dual decomposition method is an attractive alternative to solving minimum cost network flow problems when an approximation of the distribution of the optimal objective function can be used. This method can determine a close approximation of the distribution of the optimal objective function by solving a fraction of the number of problems required by full enumeration.

A close approximation of the distribution of the optimal objective function is adequate for planning purposes in most cases. The dual decomposition method allows this approximation to be determined by solving fewer problems than other methods, so it is more responsive to the time constraints of the logistics planner.

The dual decomposition method is also preferred over using expected values for arc capacity values. The expected value approach overestimates the optimal objective function and does not provide an accurate prediction on the capability of a transportation network. This overestimation on capability could produce unacceptable consequences if predictions on the logistics support available to units engaged in combat are based on the results of this method.

The dual decomposition method can also be used to make policy decisions. If the results of the analysis of a transportation network by the dual decomposition method determines that the combat units will not receive adequate logistical support, policies can be made to alleviate the problems of support.

The stockage policy on the amount of supplies available at forward supply depots could be changed to meet the demands of the combat units. If the transportation network is inadequate, stockage levels could be increased to meet the demand. If the network is capable of carrying more supplies and assets are available, the stockage level at forward depots could be decreased to reduce the susceptibility of the depots to enemy forces.

Policy concerning the amount of forces dedicated to the security of LOCs could also be made. The assignment of more security forces may decrease the probability of interdiction and thereby, increase the flow of supplies to the combat units. Security forces could also be distributed to different segments of the transportation network based on the analysis of the results determined by the dual decomposition method.

#### **B. FUTURE RESEARCH AREAS**

The dual decomposition method can be applied to other stochastic combinatorial optimization problems. These

problems find, from a finite set of alternatives, an alternative that optimizes the objective function. Examples of stochastic combinatorial optimization problems are sequencing, scheduling, and routing problems with random job durations or travel times. These problems require complete enumeration of all alternatives to find the optimal solution's distribution or moments. The dual decomposition method may be used to decrease the computational burden by limiting the number of alternatives that must be solved. This would allow combinatorial problems to be used on a more practical basis and without sophisticated computer resources.

Variations on the four day logistics problem presented in this paper should also be explored. One variation is that if an arc has several different capacities that are dependent on the weather and the probability of each different weather factor is known, the dual decomposition method could be used to determine an approximation of the optimal objective function's distribution for a maximum flow problem. Other variations are that the demands could be random rather than known and the arc capacities could be from a continuous distribution rather than discrete. The dual decomposition method could be applied to these variations as well to provide an approximate distribution of the optimal objective function without full enumeration.

A major topic that requires further research is the determination of how a threshold value of the termination criteria should be established to provide a solution within a required tolerance level. This will enable the user of the dual decomposition method to be able to establish a threshold value that is consistent with the requirements.

# APPENDIX                      COMPUTER PROGRAM

```

*****
* THIS PROGRAM GENERATES THE FOUR DAY LOGISTICS PROBLEM IN NETWORK
* FORMAT AND USES THE DUAL DECOMPOSITION METHOD TO DETERMINE
* AN APPROXIMATE OPTIMAL OBJECTIVE FUNCTION
* IT USES THE NETGEN, BUBSRT, NODPAK, AND HEDLST SUBROUTINES
*****
      INTEGER MAXDEP, MAXARC, MXNARC, MAXDAY, MXNNOD
      REAL DPINT(50,50), PINTER(50,50), PROB(100), DEST(300,-2:300)
      INTEGER NN, NA, NDA, NIGM, NDPT, NXTARC, NARC, TAIL(100),
      CHEAD(100), CAPL(200,200), CAPH(200,200),
      CPITER(100,12), DEPNUM(100), DEPNOD(100), LCAP(300), HCAP(300),
      CDCAPL(200,200), DCAPH(200,200), NH(300), THRES,
      CDAY(300), ANUM(300)
      INTEGER NXT, MXDL, DUAL, LARC, TOT, PRICE, MXDH, HARC, PNTR
*... GNET INPUT DATA
      INTEGER*4 M, T(300), H(101), C(300), CP(300), X(100)
* PRIMAL DUAL SOLUTION
      INTEGER*4 Y(300), U(100)
      PARAMETER(MXNARC = 300, MXNNOD = 100, MOLONG = 300, THRES = .5 )
      INTEGER DUAL1(MXNARC), DUAL2(MXNARC), CUR, TOT1, TOT2
      LOGICAL HLT, TERM
      CALL EXCMS( 'FILEDEF 08 DISK TH ANSWER A1 (LRECL 130 PERM)' )
*
      MAXDEP = 100
      MAXARC = 100
      MAXDAY = 12
*
      CALL NETGEN
* INPUT
      1 ( MAXDEP, MAXARC, MXNARC, MAXDAY, MXNNOD,
      2 NA, NN, NDA,
      3 NIGM, CAPL, CAPH, PINTER, DEPNUM, DCAPL, DCAPH, DPINT, DAY, ANUM,
      4 NARC, LCAP, HCAP, NH, PROB,
* OUTPUT FOR GNET
      X MNOD, T, H, C )
*
*... CONDENSE NETWORK, CREATE ARC CAPACITIES
      JX = 0
      DO 100 J=1, NARC
      IF( T(J).GT.0 ) THEN
      JX = JX + 1
      T(JX) = T(J)
      C(JX) = C(J)
      CP(JX) = HCAP(J)
      ENDIF
100 CONTINUE
      NARC = JX
*
*
*... CREATE RHS INDUCE MAX FLOW FROM CAPACITIES TO LAST NODE
      DO 110 I=1, MNOD
      X(I) = 0
110 CONTINUE
      DO 120 J=H(MNOD), H(MNOD+1)-1
      X(1) = X(1) + CP(J)
120 CONTINUE
      X(MNOD) = -X(1)
*
*
      DO 140 IH=1, MNOD
      J1 = H(IH)
      J2 = H(IH+1)-1
      DO 130 J=J1, J2
130 CONTINUE
140 CONTINUE
      IFRT = 0
      IWU = 04
      NARC = H(MNOD+1)-1
*****

```

\* SOLUTION OF THE MINIMUM COST NETWORK FLOW PROBLEM BY GNET

\*\*\*\*\*  
 \* THE DEST ARRAY IS THE QUEUE HOLDING THE PROBLEMS TO BE SOLVED  
 \*

```

CALL GNET( IPRT,IWU, MNOD,NARC, T,H,C,LCAP, X, U,Y )
DO 224 IH = 1,MNOD
  J1 = H(IH)
  J2 = H(IH + 1) - 1
  DO 225 J = J1,J2
    DUAL1(J) = U(IH) - U(T(J))
  CONTINUE
225 CONTINUE
224 DO 226 J = 1,NARC
  PRICE = C(J) * Y(J)
  TOT1 = TOT1 + PRICE
226 CONTINUE
CALL GNET( IPRT,IWU, MNOD,NARC, T,H,C,HCAP, X, U,Y )
DO 324 IH = 1,MNOD
  J1 = H(IH)
  J2 = H(IH + 1) - 1
  DO 325 J = J1,J2
    DUAL2(J) = U(IH) - U(T(J))
  CONTINUE
325 CONTINUE
324 DO 326 J = 1,NARC
  PRICE = C(J) * Y(J)
  TOT2 = TOT2 + PRICE
326 CONTINUE
CUR = 1
NXT = 1
P = 1.0
IF (TERM(TOT1,TOT2,P,THRES)) THEN
  WRITE(08,*)TOT1,TOT2,P
  CALL NOMORE
ELSE
  CALL PIVOT(DUAL1,DUAL2,PNTN,MXNNOD,NARC)
  CALL DESTOK(PNTN,DEST,DUAL1,DUAL2,NXT,CUR,MXNNOD,TOT1,TOT2,
111 1MQLONG,PROB,MXNARC,HCAP,LCAP,NARC)
  END IF
  CONTINUE
  CUR = NOHADL(CUR,MQLONG,CUR)
  IF (CUR.EQ.NXT) THEN
    CALL NOMORE
  ELSE
    END IF
    IF (DEST(CUR,-1).EQ.1) THEN
      TOT1 = DEST(CUR,-2)
      DO 200 I = 1,NARC
        DUAL1(I) = DEST(CUR,I)
        IF (DEST(CUR,I).GT.0) THEN
          CP(I) = DEST(CUR,I)
        ELSE
          CP(I) = HCAP(I)
        END IF
      CONTINUE
      CALL GNET( IPRT,IWU, MNOD,NARC, T,H,C,CP, X, U,Y )
      DO 334 IH = 1,MNOD
        J1 = H(IH)
        J2 = H(IH + 1) - 1
        DO 335 J = J1,J2
          DUAL2(J) = U(IH) - U(T(J))
        CONTINUE
      CONTINUE
      DO 336 J = 1,NARC
        PRICE = C(J) * Y(J)
        TOT2 = TOT2 + PRICE
      CONTINUE
      ELSE
        TOT2 = DEST(CUR,-2)
        DO 400 I = 1,NARC

```

```

      DUAL2(I) = DEST(CUR,I)
      IF(DEST(CUR,I).GT.0)THEN
        CP(I) = DEST(CUR,I)
      ELSE
        CP(I) = LCAP(I)
      END IF
400  CONTINUE
      CALL GNET( IPRT,IWU, MNOD,NARC, T,H,C,CP, X, U,Y )
      DO 434 IH = 1,MNOD
        J1 = H(IH)
        J2 = H(IH + 1) - 1
        DO 435 J = J1,J2
          DUAL1(J) = U(IH) - U(T(J))
635  CONTINUE
434  CONTINUE
      DO 436 J = 1,NARC
        PRICE = C(J) * Y(J)
        TOT1 = TOT1 + PRICE
436  CONTINUE
      END IF
      P = DEST(CUR,0)
      HLT = TERM(TOT1,TOT2,P,THRES)
      IF(HLT)THEN
        WRITE(4,*)TOT1,TOT2,P
      END IF
      IF(.NOT.HLT)THEN
        CALL PIVOT(DUAL1,DUAL2,PNTR,MXNNOD,NARC)
        CALL DESTOK(PNTR,DEST,DUAL1,DUAL2,NXT,CUR,MXNNOD,TOT1,TOT2,
          IMOLONG,PROB,MXNARC,HCAP,LCAP,NARC)
      ELSE
        END IF
        GOTO 111
      END
*****
* SUBROUTINE NETGEN
*****
* SUBROUTINE NETGEN
* INPUT
* 1 ( MAXDEP,MAXARC,MXNARC,MAXDAY,MXNNOD,
* 2 NA,NN,NDA,
* 3 NIGM,CAPL,CAFH,PINTER,DEPNUM,DCAPL,DCAPH,DPINT,DAY,ANUM,
* 4 NARC,LCAP,HCAP,NH,PROB,
* OUTPUT FOR GNET
* X MNOD,T,H,C )
*
* THIS SUBROUTINE READS THE NETWORK AND CONSTRUCTS THE FOUR-DAY
* PROBLEM
* DEFINITIONS: (INPUT)
* MAXARC = MAX NUMBER OF ARCS
* MAXDAY = MAX NUMBER OF DAYS
* MAXDEP = MAX NUMBER OF DEPOTS
* MXNARC = MAX NUMBER OF ARCS IN TOTAL NETWORK
* MXNNOD = MAX NUMBER OF NODES
*
* (OUTPUT)
* NA = NUMBER OF ARCS
* NN = NUMBER OF NODES
* NDA = NUMBER OF DAYS
* BIGM = A "LARGE" CAPACITY
* CAPL = LOWER CAP OF ARC
* CAFH = UPPER CAP OF ARC
* PINTER = PROBABILITY OF INTERDICTION
* DEPNUM = DEPOT NUMBER
* DCAPL = LOWER CAP OF DEPOT
* DCAPH = UPPER CAP OF DEPOT
* DPINT = PROBABILITY OF INTERDICTION (DEPOT)
* NH = HEAD
* T = TAIL
* DAY = CORRESPONDING DAY FOR THE ARC
* ANUM = ARCNUM FROM THE TABLE

```

```

*      H = HEAD ENTRY-POINT ARRAY FOR GNET
*
*      INPUT
*      NUMBER OF NODES, NUMBER OF ARCS, NUMBER OF DAYS, BIGM
*      ARC #, LOWER CAP, HI CAP, PINTER
*      NUMBER OF DEPOTS
*      DEPOT NUMBER, NODE NUMBER, LOWER CAP, HIGHER CAP, DPINT
*
*      REAL DPINT(MAXDEF,MAXDAY),PINTER(MAXARC,MAXDAY),PROB(100)
*      INTEGER NN,NA,NDA,NIGM,NDPT,NXTARC,NARC,TAIL(100),
*      CHEAD(100),CAPL(200,200),CAPH(200,200),
*      CDEPNUM(MAXDEF),DEPNOD(100),NUMSNK,
*      CDCAPL(200,200),DCAPH(200,200),NH(MXNARC),
*      CDAY(MXNARC),ANUM(MXNARC),LCAP(300),HCAP(300),
*      INTEGER LENGTH,CRIT(400),A1(400),A2(400),A3(400),
*      CA4(400),PRI,PRIOR(300,300),SINK(20)
*
*      INTEGER I,J,TMPNOD(100),ARCNUM,DEMAND,DEM(300)
*      LOGICAL SNK(300)
*      *...GNET OUTPUT DATA
*      INTEGER*4 M, T(MXNARC), H(MXNNOD+1), C(MXNARC)
*
*      CALL EXCMS('FILEDEF 03 DISK NET DATA A1')
*      CALL EXCMS('FILEDEF 04 DISK TH OUTPUT A1 (LRECL 130 PERM')
*
*      READ(03,*)NN,NA,NDA,NIGM
*
*      DO 1000 I = 1,NA
*      DO 999 J = 1,NDA
*      READ(03,*)ARCNUM,TAIL(ARCNUM),HEAD(ARCNUM),CAPL(ARCNUM,J),
*      CCAPH(ARCNUM,J),PINTER(ARCNUM,J)
999      CONTINUE
1000     CONTINUE
*      READ(03,*)NDPT
*      DO 1001 I = 1,NDPT
*      DO 998 J = 1,NDA
*      READ(03,*)DEPNUM(I),DEPNOD(I),DCAPL(I,J),DCAPH(I,J),DPINT(I,J)
998     CONTINUE
1001    CONTINUE
*
*      READ(03,*) NUMSNK
*      DO 996 I = 1,NN
*      SNK(I) = .FALSE.
996     CONTINUE
*      DO 997 I = 1,NUMSNK
*      READ(03,*)N
*      SNK(N) = .TRUE.
*      SINK(N) = I
997     CONTINUE
*
*      DO 1005 J = 1,NDA
*      DO 1004 I = 1,NA
*      T(I + ((J-1)*NA)) = (10*TAIL(I)) + J - 1
*      NH(I + ((J-1)*NA)) = (10*HEAD(I)) + J - 1
*      ANUM(I + ((J-1)*NA)) = I
*      DAY(I + ((J-1)*NA)) = J
1004    CONTINUE
1005    CONTINUE
*      NXTARC = (NDA*NA) + 1
*
*
*      DO 1007 J = 1,(NA*NDA)
*      TMPNOD(J) = (INT(T(J)/10))
1007    CONTINUE

```

```

DO 1006 I = 1,NDPT
DO 1008 J = 1,(NA*NDA)
  IF (TMPNOD(J).EQ.DEPNOD(I))THEN
    NH(NXTARC) = T(J) + 1
    T(NXTARC) = T(J) * 100
    DAY(NXTARC) = 0
    ANUM(NXTARC) = 0
    T(NXTARC+1) = T(J)
    NH(NXTARC + 1) = T(J) * 100
    DAY(NXTARC + 1) = DAY(J)
    ANUM(NXTARC+1) = -1*DEPNUM(I)
    NH(J) = NH(J) + 1
    T(J) = T(J) * 100
    DAY(J) = DAY(J) + 1
    NXTARC = NXTARC + 2
  ELSE
    END IF
1008 CONTINUE
1006 CONTINUE
DO 995 J = 1, NN
  IF (SNK(J)) THEN
    DO 994 I = 1, NDA
      T(NXTARC) = (J * 10 + 1)
      NH(NXTARC) = NIGM
      DAY(NXTARC) = I
      ANUM(NXTARC) = -1 * (J)
      T(NXTARC + 1) = 10 + I
      NH(NXTARC + 1) = J * 10 + I
      DAY(NXTARC + 1) = -I
      ANUM(NXTARC + 1) = J
      NXTARC = NXTARC + 2
    994 CONTINUE
  ELSE
    ENDIF
995 CONTINUE
  NARC = NXTARC - 1
  CALL BUBSRT(MXNARC,NARC,NH,T,NH,DAY,ANUM)
  CALL NODPAK(MXNARC,NARC,NH,T)
DO 21 J=1,NARC
21 CONTINUE
  CALL HEDLST
  1 (MXNNOD,MNOD,MXNARC,NARC,T,NH,
* OUTPUT ARRAY OF ENTRY POINTS BY HEAD NODE
  2 H)
DO 23 I=1,MNOD
23 CONTINUE
*****
* BUILD THE CAPACITY ARRAYS
*****
DO 2000 J=1,NARC
  IF((DAY(J).GT.0).AND.(ANUM(J).GT.0.AND.(DAY(J).LE.NA)))THEN
    LCAP(J) = CAPL(ANUM(J),DAY(J))
    HCAP(J) = CAPH(ANUM(J),DAY(J))
  ELSE IF(DAY(J).LE.0)THEN
    LCAP(J) = NIGM
    HCAP(J) = NIGM
  ELSE IF(DAY(J).GT.NA)THEN
    LCAP(J) = CAPL(ANUM(J),DAY(NA))
    HCAP(J) = CAPH(ANUM(J),DAY(NA))
  END IF
2000 CONTINUE
DO 2010 J=1,NARC
  IF((ANUM(J).LT.0).AND.(-1*ANUM(J).LE.NDPT))THEN
    LCAP(J) = DCAPL(-1*ANUM(J),DAY(J))
    HCAP(J) = DCAPH(-1*ANUM(J),DAY(J))
  END IF
2010 CONTINUE
*****
*
* READ IN DEMAND FOR EACH DAY
*

```

```

*****
      DO 2200 I = 1, NDA
        READ(03,*) DEMAND
        DEM(I) = DEMAND
      2200 CONTINUE
*****
*
* DO CAP ARRAY FOR DEMAND
*
*****
      DO 2001 I = 1, NARC
        IF(SNK(-1*ANUM(I))) THEN
          LCAP(I) = DEM(DAY(I))
          HCAP(I) = DEM(DAY(I))
        END IF
      2001 CONTINUE
      DO 2100 I = 1, NARC
      2100 CONTINUE
*****
* CONSTRUCT THE COST ARRAY
*
*****
      DO 2210 I = 1, NDA
        DO 2201 J = 1, NUMSNK
          READ(03,*) PRI
          PRIOR(I,J) = PRI
        2201 CONTINUE
      2210 CONTINUE
        DO 2205 I = 1, NARC
          IF(DAY(I).LT.0) THEN
            C(I) = PRIOR(-1*DAY(I), SINK(ANUM(I)))
          ELSE
            C(I) = 0
          END IF
        2205 CONTINUE
*****
*
* DO PROB ARRAY
*
*****
      DO 3000 I = 1, NARC
        IF((DAY(I).GT.0).AND.(ANUM(I).GT.0).AND.(DAY(I).LE.NDA)) THEN
          PROB(I) = FINTER(DAY(I), ANUM(I))
        ELSE IF((DAY(I).GT.NDA).OR.(DAY(I).LT.0)) THEN
          PROB(I) = 0
        ELSE IF((ANUM(I).LT.0).AND.(ANUM(I).GT.NDA)) THEN
          PROB(I) = DFINI(-1*ANUM(I), DAY(I))
        ELSE
          PROB(I) = 0
        END IF
      3000 CONTINUE
      RETURN
      END
      SUBROUTINE BUBSRT(MXNARC, LENGTH, CRIT, A1, A2, A3, A4)
*****
* SUBROUTINE BUBSRT
*
*****
      INTEGER LENGTH, CRIT(MXNARC), A1(MXNARC), A2(MXNARC), A3(MXNARC),
      CA4(MXNARC)
      INTEGER COPY(300), I, J, TEMP1, TEMP2, TEMP3, TEMP4, TEMPC
*
      DO 1000 I = 1, LENGTH
        COPY(I) = CRIT(I)
      1000 CONTINUE
*
      DO 1001 I = 1, LENGTH
        DO 1002 J = I+1, LENGTH
          IF(COPY(J).LT.COPY(I)) THEN

```

```

      TEMPC = COPY(I)
      COPY(I) = COPY(J)
      COPY(J) = TEMPC
      TEMP1 = A1(I)
      TEMP2 = A2(I)
      TEMP3 = A3(I)
      TEMP4 = A4(I)
      A1(I) = A1(J)
      A2(I) = A2(J)
      A3(I) = A3(J)
      A4(I) = A4(J)
      A1(J) = TEMP1
      A2(J) = TEMP2
      A3(J) = TEMP3
      A4(J) = TEMP4
    ELSE
      END IF
1002   CONTINUE
1001   CONTINUE
      RETURN
      END
*****
* SUBROUTINE HEDLST
*****
      SUBROUTINE HEDLST
1      ( MXNNOD,MNOD, MXNARC,NARC, T,NH,
* 2  OUTPUT ARRAY OF ENTRY POINTS BY HEAD NODE
      H )
      INTEGER MXNNOD,MNOD, MXNARC,NARC, T(MXNARC),NH(MXNARC),
      X H(MXNNOD+1)
      INTEGER ENTRY, COUNT
      MNOD = 0
      DO 100 J=1,NARC
        IF( MNOD.LT.T(J) ) MNOD = T(J)
        IF( MNOD.LT.NH(J) ) MNOD = NH(J)
100    CONTINUE
      DO 110 I=1,MNOD
        H(I) = 0
110    CONTINUE
      DO 120 J=1,NARC
        H(NH(J)) = H(NH(J)) + 1
120    CONTINUE
      ENTRY = 1
      DO 130 I=1,MNOD+1
        COUNT = H(I)
        H(I) = ENTRY
        ENTRY = ENTRY + COUNT
130    CONTINUE
      RETURN
      END
*****
* SUBROUTINE NODPAK
*****
      SUBROUTINE NODPAK( MXNARC,NARC, NH,T )
      INTEGER*4 MXNARC,NARC, NH(MXNARC),T(MXNARC), MNOD
      INTEGER J, TEMPH(100),TEMPT(100)
      LOGICAL FIRST
      DO 999 J=1,NARC
        TEMPT(J) = 0
        TEMPH(J) = 0
999    CONTINUE
*
      DO 1000 J=1,NARC
        IF( INT(T(J)/10).EQ.1 ) THEN
          TEMPT(J) = 1
        END IF
1000    CONTINUE
*
```

```

CURIDX = 2
CURNOD = NH(1)
KEEPJ = 1
*
111 FIRST = .TRUE.
DO 1001 J=KEEPJ,NARC
  IF( NH(J).EQ.CURNOD ) THEN
    TEMPH(J) = CURIDX
  ELSE
    IF( FIRST ) THEN
      FIRST = .FALSE.
      KEEPJ = J
    ENDIF
  ENDIF
CONTINUE
DO 1002 J=1,NARC
  IF( T(J).EQ.CURNOD ) TEMPT(J) = CURIDX
1002 CONTINUE
  IF( TEMPH(NARC).EQ.0 ) THEN
    CURIDX = CURIDX + 1
    CURNOD = NH(KEEPJ)
    GOTO 111
  END IF
DO 1010 J=1,NARC
  T(J) = TEMPT(J)
  NH(J) = TEMPH(J)
1010 CONTINUE
RETURN
END
*****
*
* SUBROUTINE PIVOT
*****
SUBROUTINE PIVOT(RC1,RC2,PTRL,MXNNOD,NARC)
  INTEGER RC1(MXNNOD), RC2(MXNNOD)
  INTEGER PTRL,BIG
*
  BIG = 0
  DO 100 I = 1, NARC
    IF( (ABS(RC1(I)).GT.BIG).OR.(ABS(RC2(I)).GT.BIG)) THEN
      BIG = MAX(ABS(RC1(I)),ABS(RC2(I)))
      PTRL = I
    ELSE
      END IF
100 CONTINUE
RETURN
END
*****
*
* SUBROUTINE DESTQK
*****
SUBROUTINE DESTQK(PNTR,DEST,RC1,RC2,NXT,CUR,MXNNOD,TOT1,TOT2,
  1MOLONG,PROB,MXNARC,HCAP,LCAP,NARC)
  INTEGER PNTR,NXT,CUR,RC1(MXNARC),RC2(MXNARC)
  REAL DEST(MOLONG,2:MXNARC),PROB(MXNARC)
  NXT = NOHADL(NXT,MOLONG,CUR)
  NXT1 = NOHADL(NXT1,MOLONG,CUR)
  DEST(NXT,PNTR) = LCAP(PNTR)
  DEST(NXT1,PNTR) = HCAP(PNTR)
  DO 100 I = 1,NARC
    IF( DEST(CUR,I).GT.0 ) THEN
      DEST(NXT,I) = DEST(CUR,I)
      DEST(NXT1,I) = DEST(CUR,I)
    ELSE
      DEST(NXT,I) = RC1(I)
      DEST(NXT1,I) = RC2(I)
    END IF
100 CONTINUE
  DEST(NXT,0) = PROB(PNTR)*DEST(CUR,0)

```

```

DEST(NXT1,0) = (1-PROB(PNTR))*DEST(CUR,0)
DEST(NXT1,1) = 1
DEST(NXT1,-1) = 2
DEST(NXT1,2) = TOT1
DEST(NXT1,-2) = TOT2
NXT = NXT1
RETURN
END
*****
*
*   INTEGER FUNCTION NQHADL
*
*****
INTEGER FUNCTION NQHADL(NRG,MQLONG,CUR)
INTEGER CUR,TCUR
TCUR = CUR
IF(NRG.EQ.MQLONG)THEN
  NQHADL= 1
ELSE
  NQHADL = NRG + 1
END IF
IF(NQHADL.EQ.TCUR)THEN
  PRINT*, 'QUEUE OVERFLOW AND NRG = ',NRG
  STOP
END IF
RETURN
END
*****
*
*   LOGICAL FUNCTION TERM
*
*****
LOGICAL FUNCTION TERM(TOT1,TOT2,P,THRES)
INTEGER TOT1,TOT2
REAL P,THRES
IF((ABS(TOT1-TOT2)*P).LT.THRES)THEN
  TERM = .TRUE.
  WRITE(04,*) 'TOT1 = ',TOT1,' TOT2 = ',TOT2,' P = ',P
ELSE
  TERM = .FALSE.
END IF
RETURN
END
SUBROUTINE NOMORE
STOP
END

```

## REFERENCES

1. Headquarters, Department of the Army, Field Manual 100-5, Operations, pp. 15-18.
2. United States Army Transportation School, Field Circular 55-17, Movement Control Officer's Battle Support Guide, p. 1-1.
3. Erickson, J., Hansen, L., and Schneider, W., Soviet Ground Forces - An Operational Assessment, pp. 120-121, Westview Press, 1986.
4. Bykofsky, J. and Larson, H., The Transportation Corps: Operations Overseas, Vol. 20 of United States Army in World War II, Department of the Army, 1957.
5. Wardlow, C., The Transportation Corps: Movements, Training, and Supply, Vol. 19 of United States Army in World War II, Department of the Army, 1956.
6. Ford, L.R., and Fulkerson, D.R., Flows in Networks, Princeton University Press, 1962.
7. Muster, T. M., Optimal Allocation of Air Strikes for Interdiction of a Transportation Network, M.S. Thesis, Naval Postgraduate School, Monterey, California, June 1967.
8. Preston, C.P., Interdiction of a Transportation Network, M.S. Thesis, Naval Postgraduate School, Monterey, California, March 1972.
9. Wollmer, R.D., "Algorithms For Targeting Strikes in a Lines-of-Communication Network," Operations Research, Vol. 18, pp. 497-515, May-June 1970.
10. Fishman, G.S., "The Distribution of Maximum Flow with Applications to Multistate Reliability Systems," Operations Research, Vol. 35, pp. 607-618, July-August 1987.
11. Systems Optimization Laboratory Technical Report 86-4, Reliability and Shortage Distribution Computations in General Stochastic Transportation Networks, by M.S. Bellwin, January 1986.

12. Evans, J.R., "Maximum Flow in Probabilistic Graphs - The Discrete Case," Networks, Vol. 6, pp. 161-183, January 1976.
13. The RAND Corporation Memorandum RM-55-39-PR, An Interdiction Model for Sparsely Traveled Networks, by R.D. Wollmer, April 1968.
14. Bradley, G.H., Brown, G.G., Graves, G.W., "Design and Implementation of Large Scale Primal Transshipment Algorithms," Management Science, Vol. 24, pp. 1-34, September 1977.
15. The BDM Technical Report BDM/W-80-250-TR, BDM Technical Report for the DAMSEL European Transportation Data Base, p. 17, April 1980.
16. The RAND Corporation Memorandum RM-4945-PR, An Interdiction Model of Highway Transportation, by E.P. Durbin, May 1966.

# INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93942-5002	2
3. Commandant U.S. Army Transportation School ATTN: ATSP-TDL (CPT (P) Mike Mamer) Fort Eustis, Virginia 23604-5399	2
4. Dr. Michael P. Bailey, Code 55Ba Department of Operations Research Naval Postgraduate School Monterey, California 93943	1
5. Dr. Samuel H. Parry, Code 55Py Department of Operations Research Naval Postgraduate School Monterey, California 93943	1
6. CPT Timothy P. Gannon P.O. Box 465 Martin, South Dakota 57551	1
7. Operations Analysis Programs, Code 30 Naval Postgraduate School Monterey, California 93941-5000	1
8. Commander TRADOC Analysis Command ATTN: ATRC-FA Fort Leavenworth, Kansas 66027-5200	1

- |     |  |   |
|-----|--|---|
| 9.  | Chief of Naval Operations (OP-0403)    | 1 |
|     | Department of the Navy                 |   |
|     | Washington, District of Columbia 20350 |   |
| 10. | Chief of Naval Operations (OP-09BC)    | 1 |
|     | Department of the Navy                 |   |
|     | Washington, District of Columbia 20350 |   |





Thesis  
G14397 Gannon  
c.1

The dual decomposition on  
method and its applica-  
tion to an interdicted  
network.

10 JUL 90  
19 JUL 90  
4 MAY 92

35525  
36796  
37541

Thesis  
G14397 Gannon  
c.1 The dual decomposition  
method and its applica-  
tion to an interdicted  
network.



thesG14397

The dual decomposition method and its ap



3 2768 000 81941 1

DUDLEY KNOX LIBRARY